try can be ascribed to the decay process $\omega(\Delta_1) \rightarrow 2\omega(\Lambda_3)$. As the temperature is raised, the asymmetry becomes more pronounced, until at the highest temperature (880 K) a shallow valley develops between the main resonance maximum and a subsidiary maximum on the high-frequency shoulder. At high temperatures, where three-phonon scattering processes become important, the structure of the spectral function depends critically on the detailed frequency dependence of Δ and Γ. Indeed, the minimum of the valley can be shown to correspond to a region in frequency space where $\Gamma(\omega)$ exhibits a maximum. We have, in fact, in this region $\Gamma(\omega) \gg \Gamma'(\omega) \sim 0$ and $\Delta(\omega) \sim 0$ so that $\mathrm{Im}G(\omega) \sim 1/\Gamma(\omega)$ and the spectral function directly mirrors the ω dependence of Γ. This type of structure is illustrated even more vividly in Fig. 3(c) for the longitudinal (0.15, 0.15, 0.15) mode. In this case the resonance is perfectly Lorentzian at room temperature but develops significant structure at higher temperatures, this structure being characterized by twin peaks separated by a deep minimum. Here again the minimum corresponds to a maximum in $\Gamma(\omega)$. A further interesting feature of the (0.15, 0.15, 0.15) structure is the fact that between 700 and 880 K the maximum of the resonance shifts from the left-hand peak to the right-hand peak; this effect becomes even more pronounced for $T > 880$ K. The same effect can be seen in the frequency plot of the (0.9, 0, 0) phonon. What was merely a shoulder on the low-frequency side of the resonance at 700 K becomes the actual maximum at 880 K. Two other representative phonons, the (0.5, 0.5, 0.5) and (0.5, 0.5, 0) longitudinal modes, exhibit broad shoulders on the high-frequency side.

For the phonons displayed in Figs. 3(a)–3(c), the accuracy with which the approximations of Eq. (2) predict the true position of the resonance maxima and the true half-width becomes increasingly poor as the temperature is raised. It is clear that as the structure of the resonance becomes more complex it becomes increasingly more difficult to unambiguously define the true half-width; this is especially true if the resonance develops a prominent shoulder at about half-maximum. Furthermore, the identification of the renormalized phonon frequency is not itself unambiguous, especially if the true maximum of the resonance shifts discontinuously as for the (0.9, 0, 0) and (0.15, 0.15, 0.15) phonons.

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Transient Space-Charge-Limited Currents in Photoconductor-Dielectric Structures: A Comment

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The calculation presented in an earlier paper is refined to include closed-form solutions for integrals of the exponential integral that were treated numerically in the original work. This allows us to demonstrate the equivalence of our results to those of Many and Rakavy for the limiting case of direct electrode contact to the photoconductor (no dielectric present).

In a recent paper[1] we presented the theory for transient space-charge-limited currents in photoconductor-dielectric structures. The structure was characterized by a parameter α which depended upon the relative thicknesses and dielectric constants of the two regions. It was shown that in one special limiting case ($\alpha = 1$) our general theory reproduced the results for the direct contact (no dielectric) configuration discussed by Many and Rakavy.[2] The equivalence of the two treatments was completely demonstrated, with the exception (as noted in Ref. 1) of the flow-line equation in zone II [Eq. (3.45) of Ref. 1] and consequently the equation [Eq. (3.46) of Ref. 1] which determined t_2, the time at which zone II ends. In our paper[1] we were unable to show the mathematical equivalence of these equations and the corresponding expression in Ref. 2 [Eq. (39)], primarily because of our unawareness of closed forms for various integrals of the exponential integral. However, through computation it was demonstrated that the two analyses did in fact give identical numerical results. It has recently been brought to our attention that the required integrals have been solved.[3] Therefore we carry out further reduction of Eqs. (3.45) and (3.46) of Ref. 1 and show that these newly obtained results do in fact reproduce the expressions obtained by Many and Rakavy[2] upon setting $\alpha=1$.

In our earlier work[1] we had used a somewhat unconventional definition of the exponential integral, namely, $E_1(u)\equiv\int_u^\infty (e^{-z}/z)\,dz$, $u\neq 0$. Since in this problem the lower limit on the integral is always negative, we adopt a more standard notation and define the exponential integral[3] through the relation

$$E_i(u) = -⨍_{-u}^{\infty} \frac{e^{-z}}{z}\,dz\,, \quad u>0, \tag{1}$$

where the integration symbol ⨍ denotes a Cauchy principal value. In this notation, Eq. (3.45) of Ref. 1 becomes

$$x(t) = l + \mu\left(E_p(d^+, 0^+) - \frac{\alpha V_0}{L}\right)t + \frac{2L}{\alpha}\left[\ln\left(\frac{A}{A-\ln(t/t_1)}\right) + \frac{e^{-A}}{t_1}\left[\Sigma(t, t_1) + (t-t_1)E_i(A)\right]\right], \qquad t_1 \le t \le t_2 \tag{2}$$

where $A\equiv(e^{\alpha/2}-1)^{-1}>0$,

$$\Sigma(t, t_1) \equiv \int_{t_1}^{t} dt' ⨍_{\ln(t'/t_1)-A}^{\infty} \frac{e^{-z}}{z}\,dz\,, \tag{3}$$

and other quantites have been defined previously.[1] Recall that for the physical problem considered $[\ln(t'/t_1)-A]<0$. The principal object of the present note is to complete integral (3) in a closed form.

Integral (3) can be completed by setting $[\ln(t'/t_1)-A]\equiv -u$ $(u>0)$. One obtains

$$\Sigma(t, t_1) = t_1 e^{A}\left[\int_0^{A-\ln(t/t_1)} e^{-u}E_i(u)\,du - \int_0^{A} e^{-u}E_i(u)\,du\right].$$

These integrals can be completed by using the relation[3]

$$\int_0^c e^{-u}E_i(u)\,du = \gamma + \ln(c) - e^{-c}E_i(c),$$

where γ is Euler's constant and $c>0$. The result is

$$\Sigma(t, t_1) = t_1 e^{A}\ln\left(\frac{A-\ln(t/t_1)}{A}\right) - tE_i\left(A-\ln\frac{t}{t_1}\right) + t_1E_i(A)\,. \tag{4}$$

Substituting this in (2) and simplifying, a closed-form expression for the flow lines results and is

$$x(t) = l + \mu\left(E_p(d^+, 0^+) - \frac{\alpha V_0}{L}\right)t + \left(\frac{2L}{\alpha}\,\frac{t}{t_1}\,e^{-A}\right)\times\left[E_i(A) - E_i\left(A - \frac{\ln}{t/t_1}\right)\right], \qquad t_1\le t\le t_2\,. \tag{5}$$

The equation governing t_2 is obtained by setting $t=t_2$, $x(t_2)=l$, $E_p(d^+, 0^+)=0$ in (5). One gets

$$\left(\frac{t_2}{t_1}e^{-A}\right)\left[E_i(A) - E_i\left(A-\ln\frac{t_2}{t_1}\right)\right] - \tfrac{1}{2}\alpha\frac{t_2}{t_A} = 0. \tag{6}$$

This is the closed-form analog of Eq. (3.46) of Ref. 1. It is now evident that upon setting $\alpha=1$, Eqs. (5) and (6) reproduce the corresponding expressions of Many and Rakavy.[2]

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